Price Rigidity and the Volatility of Vacancies and Unemployment *

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Abstract

The successful matching model developed by Mortensen and Pissarides seems to find its hardest task in explaining the cyclical movements of some key labor market variables such as the vacancy rate and the vacancy-unemployment ratio. Several authors have discussed mechanisms compatible with the matching technology that are able to deliver the kind of correlations observed in the data. In this paper we explore four such additional mechanisms embedded in a full blown SDGE model. We find that price rigidity greatly improves the model's empirical performance making it capable of reproducing second moments of the data. Other components such as intertemporal substitution, endogenous match destruction, capital accumulation and distortionary taxes also play a relevant role.

Keywords: unemployment, vacancies, business cycle, price rigidities
JEL Classification: E24, E32, J64.

1. Introduction

The Mortensen and Pissarides model provides an engaging explanation of the determinants of unemployment dynamics (see Mortensen and Pissarides, 1999, and the references therein). While the model has gained widespread acceptance as a theory of the

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Natural Rate of unemployment its implications for the dynamics of some key labor market variables at the business cycle frequency are less well accepted. In a widely cited paper, Shimer (2005) argues that the model is incapable of reproducing the volatility of unemployment, vacancies and the vacancy-unemployment (v/u) ratio observed in the data for a reasonable parameter calibration. This is most unfortunate, as the Mortensen and Pissarides model has become the workhorse for introducing unemployment and labor market frictions in a coherent and yet tractable way in dynamic general equilibrium models. Several authors have looked at this issue in more detail and found that the ability of the model to match data moments can be enhanced by enlarging the model in different directions (for example, Mortensen and Nagypál, 2005, Hall, 2005, or Costain and Reiter, 2005). A very promising line of research has emphasized the role of wage rigidity as a means of overcoming the shortcomings of the basic model (Bodart, Pierrard and Sneessens, 2005). In particular Gertler and Trigari (2005) forcefully argue that nominal wage stickiness in the form of a Calvo (1983) adjustment process of the Nash bargaining wage moderates the volatility of real wages making labor market variables more volatile.

In this paper we take an alternative stance and approach the issue in a complementary way. Like Gertler and Trigari (2005) and den Haan, Ramey and Watson (2000), we argue that the model performance at business cycle frequency can be greatly improved by embedding the basic search and matching model in a broader general equilibrium framework, but we stick to the assumption of wage flexibility and explore other mechanisms instead, namely, endogenous separation rates, price rigidity, intertemporal substitution, capital and taxes. These seemingly unrelated features may have different or even offsetting effects on the model's capability to match the data but have, nonetheless, something in common: they all bring the model closer to a state-of-the-art SDGE model and thus provide a richer framework to assess the usefulness of the search and matching structure to explain the data. Besides, each of these mechanisms is relevant on its own. Endogenous separation seems the right choice if we want to give firms an additional margin with which to optimize and adjust employment in the presence of technology shocks. Price rigidity might contribute to smoothing out the response of real wages. Real interest rate fluctuations affect the present value of future surpluses. Capital accumulation is a key component of a model of business cycle fluctuations whose interaction with the labor market cannot be ignored. Finally, distortionary taxes influence the response of investment and the net values of surpluses, thus affecting unemployment and vacancies.

Our main result is that price rigidity is critical for the model to deliver the kind of volatility observed in vacancies and the unemployment vacancy ratio. We see price rigidity as mechanism akin to that of wage stickiness. Following positive supply shocks, rigid prices generate large swings in the mark-up of those firms posting vacancies and then in the surplus of matches. As the expected value of a match increases so do vacancies, thus leading to higher volatility than under flexible prices. Other components of the model play a relevant but lesser role in quantitative terms. First, as expected, endogenous separation does reduce the volatility of vacancies; instead of posting new vacancies firms may reduce the amount of endogenous destruction, thus making vacancies less sensitive to technology shocks. Second, the degree of intertemporal substitution affects the fall in the real interest rate induced by a positive technology shock, thus affecting the value of a vacancy. Third, when capital and labor are complementary inputs, the presence of capital as a production factor significantly strengthens the response of the surplus to technology shocks and the opening of new jobs beomes more attractive. Finally, distortionary taxes also increase the volatility of both unemployment and vacancies.

The rest of the paper is organized as follows. In section II we outline a general version of the model used in the paper. In section III we present the empirical evidence and discuss the calibration in detail. Section IV presents the main results summarized above and section V concludes.

2. The model

There are three types of agents in this economy: firms, workers and the government. The model embeds Mortensen and Pissarides trading technology in the labor market into a fairly general equilibrium model with capital and sticky prices.

2.1 Households

Households maximize the β discounted present value of the following utility function,

$$\overline{U}_{it}\left(c_{it}^{*}, A_i\right) = U\left(c_{it}^{*}\right) - \chi_{it} A_i \tag{1}$$

where:

$$U_i\left(c_{it}^{*}\right) = \frac{\left(c_{it}^{*}\right)^{1-\sigma}}{1-\sigma} \tag{2}$$

$$c_{it}^{*} = \frac{c_{it}}{c_{it-1}^{h}} \tag{3}$$

and h is a parameter that if different from zero indicates the presence of consumption habits, A_i stands for the disutility of working with $\chi_i = 1$ if the worker is employed and 0 otherwise. The budget constraint is given by

$$(1+\tau_t^c)\,c_{it}+e_{it}+\frac{M_{it}}{P_t}+\frac{B_{it}}{P_t}=\left[\begin{array}{c}\chi_{it}y_{it}^l+\left(1-\tau_t^k\right)r_t k_{it-1}+\\ \frac{M_{it-1}}{P_t}+(1+i_{t-1})\,\frac{B_{it-1}}{P_t}+\int_0^1\frac{\Omega_{ijt}}{P_t}dj\\ +(1-\chi_{it})\,\widetilde{g}_t^u+g_t^s+\frac{M_{it}^s}{P_t}\end{array}\right] \tag{4}$$

where c_{it} stands for consumption, e_{it} for investment, M_{it} are money holdings, B_{it} bond holdings, r_t the real return on capital, i_t nominal interest rate and Ω_{ij} is the share of profits from the jth retail sector firm that flows to household i. $\widetilde{g}^u$ is the unemployment benefit, g_i^s is a lump sum transfer from the government, k_{it-1} is the stock of capital at the end of period $t-1$ held by household i, y_{it}^l represents household's disposable real labor income (see definition below) and M_{it}^s the monetary transfers from the government (in aggregate $M_t^s=(M_t-M_{t-1})$). The model has taxes on capital (τ_t^k) and labor (τ_t^w) incomes, and consumption (τ_t^c).

Money is required to make transactions,

$$P_t\left(1+\tau_t^c\right)c_{it}\leq M_{it-1}+M_{it}^s \tag{5}$$

and households accumulate capital for which they have to pay installation costs ϕ_t and then rent it to firms at the rental cost r_t

$$k_{it}=(1-\delta)\,k_{it-1}+\phi_t k_{it-1} \tag{6}$$

where $\phi_t=\phi\left(\frac{e_{it}}{k_{it-1}}\right)$. We further assume that households are homogenous and that they pool their incomes at the end of the period. Symmetry in consumption allows us to drop the i subscript for the remainder of the paper.

The first order conditions for an interior solution are:

$$\frac{c_t^{-\sigma}}{c_{t-1}^{h(1-\sigma)}}-E_t\beta h\frac{c_{t+1}^{1-\sigma}}{c_t^{h(1-\sigma)+1}}-\lambda_{1t}\left(1+\tau^c\right)-\lambda_{2t}\left(1+\tau^c\right)=0 \tag{7}$$

$$\lambda_{1t}-\lambda_{3t}\phi'=0 \tag{8}$$

$$\begin{array}{c}E_t\beta\lambda_{1t+1}\left(1-\tau_{t+1}^k\right)r_{t+1}-\lambda_{3t}+\\ E_t\beta\lambda_{3t+1}\left[(1-\delta)+\phi_t-\phi_t'\frac{e_{t+1}}{k_t}\right]=0\end{array} \tag{9}$$

$$\lambda_{1t}-E_t\beta\lambda_{1t+1}\frac{P_t}{P_{t+1}}-E_t\beta\lambda_{2t+1}\frac{P_t}{P_{t+1}}=0 \tag{10}$$

$$\lambda_{1t}-E_t\beta\lambda_{1t+1}\left(1+i_t\right)\frac{P_t}{P_{t+1}}=0 \tag{11}$$

where λ_{1t+1} is the Lagrangian multiplier associated to the budget constraint, λ_{2t+1} is the Lagrangian multiplier associated to the CIA constraint and λ_{3t+1} is the Lagrangian multiplier associated to the law of motion of capital. From (10) and (11) we obtain that,

$$\lambda_{2t} = i_{t-1}\lambda_{1t} \tag{12}$$

Inserting (12) in (7):

$$\lambda_{1t} = \frac{\frac{c_t^{-\sigma}}{c_{t-1}^{h(1-\sigma)}} - \beta E_t h \frac{c_{t+1}^{1-\sigma}}{c_t^{h(1-\sigma)+1}}}{\left(1+\tau_t^c\right)\left(1+i_{t-1}\right)} \tag{13}$$

now from (11) the Euler equation is derived,

$$\lambda_{1t}\beta^{-1} = \left(1+i_t\right)E_t\left(\lambda_{1t+1}\frac{P_t}{P_{t+1}}\right) \tag{14}$$

and from (8) we obtain Tobin's q,

$$\frac{\lambda_{3t}}{\lambda_{1t}} = \left[\phi_t'\right]^{-1} = q_t \tag{15}$$

finally from (9),

$$q_t\beta^{-1}=E_t\left\{\frac{\lambda_{1t+1}}{\lambda_{1t}}\left(\left(1-\tau_t^k\right)r_{t+1}+q_{t+1}\left[\left(1-\delta\right)+\phi_t-\phi_t'\frac{e_{t+1}}{k_t}\right]\right)\right\} \tag{16}$$

2.2 The retail sector

Households and the government demand a single final good y_t for consumption and investment. y_t is a composite of different varieties produced by monopolistically competitive retail firms with elasticity of substitution θ. Each $\widetilde{j}_{th}$ retail firm buys the output of wholesale firms at the price P_t^w; this price is common to all wholesale firms since that sector is competitive. Then retail firms convert this output in a variety $y_{\tilde{j}t}$ that is sold in the market at the price $P_{\tilde{j}t}$. The demand for each variety is given by

$$y_{\tilde{j}t} = \left(\frac{P_{\tilde{j}t}}{P_t}\right)^{-\theta} y_t \tag{17}$$

and the aggregate price is given by:

$$P_t = \left[\int_0^1 \left(P_{\widetilde{j}t} \right)^{1-\theta} d\widetilde{j} \right]^{\frac{1}{1-\theta}} \tag{18}$$

Prices are sticky in the retail sector. Following Calvo (1983), each period a random fraction of firms adjust prices. Let P_t^* be the optimal price of the representative firm changing prices at t and $(1-\omega)$ the probability a firm adjusting prices satisfying

$$P_t^* = \left(\frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t,t+s} \left[\mu_{t+s}^{-1} \left(P_{t+s} \right)^{\theta+1} y_{t+s} \right]}{E_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t,t+s} \left[\left(P_{t+s} \right)^{\theta} y_{t+s} \right]} \tag{19}$$

where μ_t represents the mark$-$up $\left(\frac{P_t}{P_t^w} \right)$ that firms in the retail sector face, i.e. the inverse of the marginal cost, and $\Lambda_{t,t+s}$ is the firm's discount rate, i.e., the pricing kernel that must satisfy the following condition:

$$\frac{E_t \Lambda_{t,t+s}}{E_t \Lambda_{t,t+s-1}} = \frac{E_t(\lambda_{1t+s}/P_{t+s})}{E_t(\lambda_{1t+s-1}/P_{t+s-1})} \tag{20}$$

The aggregate price level in the retail price sector is given by,

$$P_t = \left[\omega P_{t-1}^{1-\theta} + (1-\omega) \left(P_t^* \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{21}$$

2.3 Intermediate sector

Firms in the intermediate perfectly competitive wholesale sector carry out the actual production and hire labor. Each producing firm employs one worker and technology is given by,

$$\widetilde{y}_{jt} = z_t a_{jt} k_{jt}^{\alpha} \tag{22}$$

where k_{jt} is the amount of capital (capital-labor ratio) optimally decided by the firm, z_t is a common aggregate autocorrelated shock with parameter ρ_z and a_{jt} is a firm specific productivity shock. Both shocks have a mean of 1. Nominal income at t is $P_t^w \widetilde{y}_{jt}$ but only becomes available in period $t+1$; thus, real income is given by $\frac{P_t^w}{P_{t+1}} \widetilde{y}_{jt}$. Present value real income is given by,

$$\left(\frac{1}{1+i_t} \right) \frac{P_t^w}{P_t} \widetilde{y}_{jt} = \left(\frac{1}{1+i_t} \right) \frac{z_t a_{jt} k_{jt}^{\alpha}}{\mu_t} \tag{23}$$

where we have made use of the appropriate discount factor obtained from (11),

$$\beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}} \frac{P_t}{P_{t+1}} \right) = \frac{1}{1+i_t} \equiv \frac{1}{R_t} \tag{24}$$

Let us normalize the population to 1. Matching and production take place in period t. At the beginning of period t some workers and firms are matched while others are not. In particular, workers start period t either matched (n_t) or unmatched $(1 - n_t)$. Some of these matches are destroyed throughout this period while others are created. Unmatched firms and those whose match is severed at that period decide whether or not to post a vacancy. This decision is studied later. Posted vacancies are visited randomly by unemployed workers and all visited vacancies are occupied so that a new match occurs.

In period t not all matches become productive. Before production takes place there is an exogenous probability ρ^x of match finishing, so only $(1 - \rho^x)n_t$ matches survive this exogenous selection. Surviving matches observe the realization of the random firm specific productivity shock a_{it}. If a_{it} is larger than some (endogenous) threshold $\widetilde{a}_t$ then the match becomes a productive firm, otherwise ($a_{it} < \widetilde{a}_t$) the match is severed with probability

$$\rho^n_{jt} = I(\widetilde{a}_t) = \int_{-\infty}^{\widetilde{a_{jt}}} \varphi(a_{jt}) da_{jt} = I\left(\widetilde{a}_{jt}\right) \tag{25}$$

so the (match specific) survival rate is given by $\rho^s_{jt} = \left(1\text{-}\rho_{jt}\right) = \left(1\text{-}\rho^x\right)\left(1\text{-}I\left(\widetilde{a}_{jt}\right)\right)$ where $\rho_{jt} = \rho^x + (1 - \rho^x)\rho^n_{jt}$ is the proportion of matches that do not survive.

The unemployment rate is given by $u_t \equiv (1-n_t) + \rho_t n_t$ workers are unemployed during this period. Notice that employment and unemployment do not add up to 1 since the unemployment rate we consider here is neither the beginning nor the end of period rate but rather the amount of workers that have been unemployed at some point during period t. Unemployed workers are actively looking for vacancies that will eventually become productive (if they ever do) in $t + 1$.

The number of new matches in period t is ϑ, so that the total number of matches evolves according to:

$$n_{t+1} = (1 - \rho_t)n_t + \vartheta \tag{26}$$

The number of matches in period t depends on the amount of vacancies posted and unemployed workers looking for jobs. The mapping from u_t and v_t into the number of matches is given by an aggregate matching function $\vartheta(u_t, v_t)$. The probability of a worker finding a job is given by

$$\rho_t^w = \frac{\vartheta(u_t, v_t)}{u_t} \tag{27}$$

and similarly, the probability of firm with a posted vacancy actually finding a match is

$$\rho_t^f = \frac{\vartheta(u_t, v_t)}{v_t} \tag{28}$$

Let us look at the choices taken throughout this process in more detail. When a vacancy is visited the job offer is accepted and if the match is not severed, either because of exogenous or endogenous separation, it produces $\widetilde{y}_{jt}$. The joint payoff of this match is

$$\left[\left(\frac{1}{1+i_t}\right)\frac{z_t a_{jt} k_{jt}^{\alpha}}{\mu_t} - r_t k_{jt}\right] - A + x_{jt} \tag{29}$$

where x_t is the expected current value of future payoffs obtained if the relationship continues into the next period. A match continues if the expected payoff (29) compensates for the loss of alternative opportunities available to firms, government and workers. There are no alternative opportunities for firms or the government and the alternative opportunities for workers is the value if unemployed w_{jt}^u, where $w_{jt}^u = \widetilde{g}^u + \overline{w_{jt}^u}$, $\overline{w_{jt}^u}$ is the present value of future worker opportunities if unemployed in period t to be defined below and $\widetilde{g}^u$ represents unemployment compensation.

The threshold specific shock $\widetilde{a_{jt}}$ below where production is not undertaken satisfies

$$\left[\frac{z_t \widetilde{a}_{jt}\left(\widetilde{k}_{jt}^{*}\right)^{\alpha}}{(1+i_t)\mu_t} - r_t \widetilde{k}_{jt}^{*}\right] - A + x_{jt} - w_{jt}^u = 0 \tag{30}$$

that is evaluated at $\widetilde{k}_{jt}^{*}$ that represents the optimal value of capital had $\widetilde{a_{jt}}$ occurred. This optimal capital (labor ratio) is given by:

$$\widetilde{k}_{jt}^{*} = \left(\frac{\alpha z_t \widetilde{a}_{jt}}{(1+i_t)\mu_t r_t}\right)^{\frac{1}{1-\alpha}} \tag{31}$$

If production takes place the firm chooses its capital optimally to

$$\max_{k_{jt}} \left[\left(\frac{1}{1+i_t}\right)\frac{z_t a_{jt} k_{jt}^{\alpha}}{\mu_t} - r_t k_{jt}\right] - A + x_{jt} \tag{32}$$

$$\frac{\alpha z_t a_{jt} k_{jt}^{\alpha-1}}{(1+i_t)\mu_t} - r_t = 0 \rightarrow k_{jt}^* = \left(\frac{\alpha z_t a_{jt}}{(1+i_t)\mu_t r_t}\right)^{\frac{1}{1-\alpha}} \tag{33}$$

Define $x_t^u = x_t - w_t^u$ as the expected excess value of a match that continues into period $t+1$ and s_{jt+1} as the joint surplus of a match at the start of $t+1$, then for the optimal capital

$$s_{jt+1}^* = \left[\left(\frac{1}{1+i_{t+1}}\right)\frac{z_{t+1}a_{jt+1}\left(k_{jt+1}^*\right)^\alpha}{\mu_{t+1}} - r_{t+1}k_{jt+1}^*\right] - A + x_{jt+1}^u \tag{34}$$

Wage is determined as a result of a Nash bargaining process whereby the surplus is split among the worker and the firm according to the relative bargaining power of each side. In particular a proportion η of the surplus will be received by the worker, who pays $\tau_{t+1}^w \eta s_{jt+1}^*$ in taxes, while the firm receives $1-\eta$ of the match surplus. Hence total after tax labor income is given by

$$y_{jt+1}^l = \left(1 - \tau_{t+1}^w\right)\left[\eta s_{jt+1}^* + A - x_{jt+1}^u\right] \tag{35}$$

while the government receives a total of *per* match:

$$\tau_{t+1}^w \left(\eta s_{jt+1}^* + A - x_{jt+1}^u\right) \tag{36}$$

The firm will receive $(1-\eta)\, s_{jt+1}^* + r_{t+1}k_{jt+1}^*$ that is used to pay the rental cost of capital and vacancy posting costs. Total production can be obtained by adding up total rents.

An unemployed worker in t finds a match with probability ρ_t^w. With probability $1-\rho_t^w(1-\rho_{t+1})$ the worker either fails to make a match or makes a match that does not produce in $t+1$. In either case the worker only receives w_{t+1}^u. The expected discounted value net of taxes for an unmatched worker, and hence her relevant opportunity cost of being matched, is:[2]

$$w_{jt}^u = \widetilde{g}^u + \beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}}\right)\left[\rho_t^w (1-\rho^x)\int_{\widetilde{a}_{t+1}}^{a_{\max}} \left(1-\tau_{t+1}^w\right)\eta s_{jt+1}^* \varphi(a_i) da_i + w_{jt+1}^u\right] \tag{37}$$

For an existing match, the match produces in $t+1$ with probability $1-\rho_{t+1}$. In this case the worker will receive y_{t+1}^l net of taxes. For a worker and firm already matched

[2] Note that recursivity in equation (37) implies a permanet flow of income from $\widetilde{g}^u$ that should be taken into account in the calibration.

the joint discounted value of an existing match is $(1\text{-}\eta)\, s^*_{jt+1} + \left(1\text{-}\tau^w_{t+1}\right) \eta s^*_{jt+1} + w^u_{t+1}$, with probability $1 - \rho_{t+1}$, and w^u_{t+1}, with probability ρ_{t+1}. This allows us to write the expected current value of future payoffs of an existing match as:

$$x_{jt} = \beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}}\right) \left[(1-\rho^x) \int_{\widetilde{a}_{t+1}}^{a_{\max}} \left(1 - \eta\tau^w_{t+1}\right) s^*_{jt+1}\varphi(a_j)da_j + w^u_{jt+1}\right] \tag{38}$$

Therefore:

$$x^u_{jt} \equiv x_{jt} - w^u_{jt} = \beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}}\right) (1-\rho^x) \left[1 - \eta\rho^w_t - \eta\tau^w_{t+1}\left(1-\rho^w_t\right)\right] \int_{\widetilde{a}_{t+1}}^{a_{\max}} s^*_{jt+1}\varphi(a_j)da_j - \widetilde{g}^u \tag{39}$$

Unmatched firms or those whose matches terminated may enter the labor market and post a vacancy. Posting a vacancy costs γ per period and the probability of filling a vacancy is ρ^f_t. Free entry ensures that

$$\beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}}\right) \rho^f_t (1-\rho^x) \int_{\widetilde{a}_{t+1}}^{a_{\max}} (1-\eta) s^*_{jt+1}\varphi(a)da = \gamma \tag{40}$$

hence

$$x^u_{jt} = \frac{\gamma\left[1 - \eta\rho^w_t - \eta\tau^w_{t+1}\left(1-\rho^w_t\right)\right]}{\rho^f_t\,(1-\eta)} - \widetilde{g}^u \tag{41}$$

Equilibrium in the capital market is determined by the following market−clearing condition:

$$(1-\rho^x)\, n_t \int_{\widetilde{a}_t}^{\infty} k^*_{jt}\varphi(a_t)\, da_t = k_{it-1} \tag{42}$$

where the left hand side indicates the demand for capital to produce in t and the right hand side is the supply of capital available to produce in t derived from (7) to (11).

For aggregation we have to take into account that the specific shock can be different for each firm. Thus, aggregation requires a double integral, one for all possible realizations of the specific shock and the other for all firms that actually produce. The result of the latter integral gives the number of active matches $(1-\rho_t)n_t$, whereas the former integral can be interpreted as the average realization of the shock. Therefore aggregate output net of vacancy costs of the wholesale sector is obtained from:

$$y_t = (1-\rho_t)n_t z_t \int_{\widetilde{a}_t}^{a_{\max}} a_t k^*_{jt} \frac{\varphi(a)}{1 - I\left(\widetilde{a}_t\right)} da - \gamma v_t \tag{43}$$

or:

$$y_t=(1-\rho^x)n_t z_t\left(\frac{\alpha z_t}{(1+i_t)\,\mu_t r_t}\right)^{\frac{\alpha}{1-\alpha}}\int_{\widetilde{a}_t}^{a_{\max}} a^{\left(\frac{1}{1-\alpha}\right)}\varphi(a)da-\gamma v_t \tag{44}$$

where we have considered that the distribution function for a_j is common across firms and independent over time. Define

$$k_t^* = \int_{\widetilde{a}_t}^{a_{\max}} k_{jt}^* \frac{\varphi(a)}{1-I(\widetilde{a}_t)}da \tag{45}$$

the average optimal capital, and define

$$s_{t+1}^* = \int_{\widetilde{a}_{t+1}}^{a_{\max}} s_{jt+1}^* \frac{\varphi(a)}{1-I(\widetilde{a}_{t+1})}da \tag{46}$$

the average joint surplus of the match at the start of $t+1$. Given that the expected excess value of a match is equal for all matches, we can suppress subindex j and write x_{jt}^u as x_t^u

2.4 Government

Tax revenues are defined as:

$$t_t = \tau_t^c c_t + \tau_t^k r_t k_{t-1} + \tau_t^w (1-\rho_t) n_t \left(\eta s_t^* + A - x_t^u\right) \tag{47}$$

The budget constraint in real terms for the government is defined by:

$$\frac{M_t}{P_t} + \frac{B_t}{P_t} = (1+i_{t-1})\,\frac{B_{t-1}}{P_t} = g_t^c + g_t^s + g^u u_t + \frac{M_{t-1}}{P_t} + \frac{M_t^s}{P_t} - t_t \tag{48}$$

where g_t^c represents public consumption. Define $b_t = \frac{B_t}{P_t}$ and $\pi_t = \frac{P_t}{P_{t-1}}$. Given the definition in aggregate for M_t^s is reduced to:

$$b_t - (1+i_{t-1})\,\frac{b_{t-1}}{\pi_t} = g_t^c + g_t^s + g^u u_t - t_t \tag{49}$$

To close the model it is necessary to specify both a policy rule and a monetary rule. The fiscal rule reflects the adjustment in public consumption to deviation from a debt objective:

$$g_t^{\varphi} = g_{t-1}^{\varphi} + \psi_1^{\varphi}\left[\overline{\left(\frac{b}{y}\right)} - \left(\frac{b_t}{y_t}\right)\right] + \psi_2^{\varphi}\left[\left(\frac{b_{t-1}}{y_{t-1}}\right) - \left(\frac{b_t}{y_t}\right)\right] \tag{50}$$

where φ stands for superscript c, s

There is a Taylor rule setting the nominal interest rate as a function of deviation of inflation with respect to a target inflation rate $\overline{\pi_t}$:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\left[\rho_\pi(\pi_t - \overline{\pi_t}) + \rho_y (y_t - \overline{y}) + \overline{i}\right] \tag{51}$$

3. Calibration

In order to analyze the main quantitative implications of our model, we have obtained a numerical solution of the steady state as well as of the log-linearized system (see Appendixes 1 to 3).

Parameter values are chosen so that the baseline solution replicates the steady state U.S. economy. The calibrated parameters and exogenous variables appear in Table 1 and the implied steady state in Table 2. The calibration strategy begins by solving for $\overline{\rho}$, $\overline{u}$, $\overline{v}$, $\overline{\overline{a}}$ and ν_0 (the scale parameter in the matching function) using the steady-state equations (see Appendix 2). To obtain these five unknown variables we need to set the steady state values of some endogenous variables. Thus $\overline{n}$ has been set to the sample average, 0.9433 and the mean quarterly separation rate is approximately 0.09 (Hall, 2005). Given that $\overline{n} = 0.9433$ and $\overline{\rho} = 0.09$ imply that in steady-state the average rate of workers looking for a job within each quarter is $\overline{u} = 0.142$, then $\overline{\rho n} = \overline{u \rho}^w$ implies a value of $\overline{\rho}^w$ equal to 0.6. This value of $\overline{\rho}^w$, which applies to $\overline{u}$, is consistent with a value of 1.479 of the quarterly job-finding rate which applies to the average US unemployment rate, slightly higher than the value of 1.35 estimated by Shimer (2005). Also form the steady-state condition $\overline{\rho}^f\overline{v} = \overline{\rho}^w\overline{u}$ and using data from JOLTS in which the average 2001:1 to 2004:3 ratio $\overline{v}/(1-\overline{n})$ equals 0.58, we derive $\overline{\rho}^f = 2.58$, which implies that a vacancy is open on average 5 weeks. We assume that $\rho^x = 0.072$ that implies that the exogenous separation rate is 80 per cent of the total separation rate, a number between the one assumed by den Haan et al. (2000) but smaller than the one implied by Hall (2005) who suggests that the total separation rate is almost completely acyclical. Finally, we assume that a follows a log normal distribution with standard deviation of 0.10, the same as den Haan et al. (2000). We set the share of the match surplus that the worker receives (η) equal to $2/3$, between 0.5 (Walsh, 2005) and 0.72 (Shimer, 2005). With these numbers equations (2.1) to (2.5) imply that $\widetilde{a} = 0.8133$ and $\nu_0 = 1.075$.

The preference parameters are set to conventional values. In particular we take from Walsh (2005) the following parameters: the discount rate (β), the risk aversion (σ), the elasticity of demand of differentiated goods (θ) and habits (h). Government consumption ($\overline{g}^c/\overline{y}$), investment ($\overline{g}^p/\overline{y}$) and transfers over GDP ($\overline{g}^s/\overline{y}$) are set to historical average values. Capital and consumption tax rates have been taken from Boscá, García and Taguas (2005), whereas τ^w has been calibrated to obtain a debt to GDP ratio equal to 2 at quarterly frequency. For simplicity, unemployment benefits are assumed to be equal to the replacement rate times the average labor income:

$$\overline{g}^u = \overline{rr}\frac{\overline{y}^l}{\overline{n}} \tag{52}$$

where $\overline{rr} = 0.26$, taken from the average value from1960 to 1995 in Blanchard and Wolfers (2000). We calibrate $\overline{g}^s$ using the following steady-state relationship:

$$\frac{g^u\overline{u} + \overline{g}^s}{\overline{y}} = \frac{\overline{rr}\frac{\overline{y}^l}{\overline{n}}\overline{u} + \overline{g}^s}{\overline{y}} = 0.15 \tag{53}$$

and hence:

$$\frac{\overline{g}^s}{\overline{y}} = 0.15 - \overline{rr}\frac{\overline{y}^l}{\overline{yn}}\overline{u} \tag{54}$$

The elasticity of output to private capital (α) is 0.4 and the depreciation rate (δ) is 0.02. Capital adjustment costs are assumed to satisfy the following properties: $\phi^{-1}(\delta)$ $= \delta$ and $\phi'\left(\frac{\overline{e}}{\overline{k}}\right) = 1$. Therefore, in steady state equation (2.9) implies $\overline{q} = 1$, which allows equations (2.19) and (2.8) to be rewritten as:

$$\overline{e} = \delta\overline{k} \tag{55}$$

and

$$1 = \beta\left(1 - \overline{\tau}^k\right)\overline{r} + \beta(1-\delta) \tag{56}$$

Capital adjustment costs ($\Phi = \phi''(\overline{e/k})$) are equal to -0.25 as in Bernanke, Gertler and Gilchrist (1999).

Since the discount factor (β) is 0.989, following Christiano and Eichenbaum (1992), equation (2.7) implies a steady-state value of $\overline{i}$

$$\overline{i} = \frac{\overline{\pi}}{\beta} - 1 = 0.011 \tag{57}$$

The rental cost of capital is obtained from expression (56)

$$\overline{r} = \frac{1 - \beta(1-\delta)}{\beta\left(1 - \overline{\tau}^k\right)} \tag{58}$$

The elasticity of demand for the differentiated retail goods is set equal to 11, as in den

Haan et al. (2000) implying a steady state mark-up value of 1.1:

$$\overline{\mu} = \frac{\theta}{\theta - 1} \tag{59}$$

The values of $\overline{i}$, $\overline{r}$ and $\overline{\mu}$ can be plugged in equation (2.11) to obtain the steady state value for the optimal individual capital demand

$$\overline{k}^{*} = \left(\frac{\alpha \delta z a}{\left(1 + \overline{i}\right) \overline{\mu r}} \right)^{\frac{1}{1-\alpha}} \tag{60}$$

given the steady-state values of $\overline{n}$, $\overline{k}^{*}$ and $\widetilde{a}$, we can use equation (2.12) to obtain the aggregate capital stock

$$\overline{k} = (1 - \rho^{x}) \, \overline{n} \overline{k}^{*} \left(1 - I\left(\widetilde{a}\right)\right) \tag{61}$$

whereas equation (2.13) gives $\overline{k}_i$. Using the steady-state values of $\overline{\mu}$, $\overline{k}^{*}$, $\widetilde{a}$, $\overline{r}$ and $\overline{i}$, equation (2.14) can be solved for A allowing us to calibrate γ to satisfy equation (2.17). Then, given $\overline{n}$, $\overline{k}^{*}$, $\widetilde{a}$ and γ, the production function gives the steady-state value of output net of vacancy costs, which are equal to 0.6 per cent of GDP.

Since the steady-state investment is given by equation (55), the aggregate resource constraint allows to obtain private consumption $\overline{c}$, making it possible to solve for $\overline{\lambda}$ in expression (2.21) and $\overline{m}$ in expression (2.22). Finally, $\overline{x}^{u}$, $\overline{s}^{*}$, $\overline{y}^{l}$, $\overline{t}$ and $\overline{b}$ can be solveb recursively in equations (2.15) to (2.25).

Some relevant parameters cannot be obtained from the steady-state relationships. Thus, we adopt a value of 0.7 for ω close to empirical estimates of the average duration of price stickiness (Gali and Gertler, 1999, Sbordone, 2002), whereas for inflation indexation we take an intermediate value ($\varsigma = 0.5$). As regards the fiscal policy, we assume that only transfers respond to debt deviations from the target so that the dynamics of the all others variables are unaffected; this implies that ψ_1^s is the only parameter of the fiscal rule initially set different to zero. The parameters in the interest rule are standard in the literature: $\rho_i = 0.75$, $\rho_\pi = 1.50$ and $\rho_y = 0$. Finally the standard deviation of productivity shocks (σ_z) and its autocorrelation parameter (ρ_z) are calibrated to reproduce the average historical volatility and autocorrelation of the US output gap.

The model with transitory supply shocks (that is, shocks in z_t) has been simulated 1000 times, with 260 observations in each simulation. We take the last 160 quarters and compute the averages over the 1000 simulations of the standard deviation of each variable (x) relative to that of output (σ_x/σ_y, except for GDP which is just σ_y), the first-order autocorrelation (ρ_x) and the contemporaneous correlation with output (ρ_{xy})

TABLE 1 — PARAMETER VALUES

ν_0	1.075	γ	0.500	ω	0.700
ρ^x	0.072	h	0.780	ς	0.500
β	0.989	$\overline{g}^c/\overline{y}$	0.150	Φ	-0.25
δ	0.020	$\overline{g}^s/\overline{y}$	0.141	ρ_i	0.750
θ	11	$\overline{g}^p/\overline{y}$	0.035	ρ_π	1.500
$\alpha = \nu$	0.400	τ^w	0.345	ρ_y	0.000
$\overline{rr}$	0.300	τ^k	0.350	σ_a	0.100
σ	2.000	τ^c	0.100	σ_z	1.700
A	1.524	η	0.666	ρ_z	0.402

TABLE 2 — STEADY STATE

$\overline{\rho}$	0.090	$\overline{r}$	0.048	$\overline{\lambda}$	0.078
$\overline{u}$	0.141	$\overline{q}$	1.000	$\overline{m}/\overline{y}$	0.731
$\overline{v}$	0.033	$\overline{\mu}$	1.100	$\overline{x}^u/\overline{y}$	0.017
$\widetilde{a}$	0.813	$\overline{k}^*/\overline{y}$	8.804	$\overline{s}^*/\overline{y}$	0.193
$\overline{n}$	0.943	$\overline{k}/\overline{y}$	7.557	$\overline{b}/\overline{y}$	2.000
$\overline{\rho}^f$	2.581	$\overline{y}$	1.000	$\overline{k}_i/\overline{y}$	5.451
$\overline{\rho}^w$	0.600	$\overline{e}/\overline{y}$	0.151		
$\overline{i}$	0.011	$\overline{c}/\overline{y}$	0.664		

of each variable.

These moments are compared with basic labor markets facts of the US business cycles from 1951:1 to 2005:3. The data source is basically the same as in Shimer (2005). We use FRED Economic Data from the Federal Reserve Bank of St. Louis for unemployment, the help wanted index and civilian employment . As the frequency of these data is monthly, we compact the data set by taking quarterly averages. The real quarterly GDP (billions of chained 2000 dollars) is obtained from the Bureau of Economic Analysis of the Department of Commerce. We take logs of these quarterly variables and obtain their cyclical components using the Hodrick-Prescott filter with a smoothing parameter equal to 1600.[3]

[3] We have checked that we obtain the same results as in Shimer (2005) if the analysed period dates from 1951:1 to 2003(4) and the smoothing parameter is 100000.

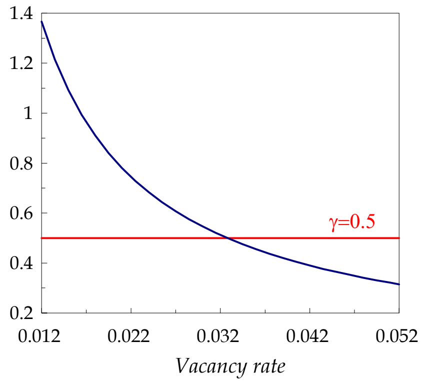

Figure 1: *Free entry condition.*

4. Results

The results discussed in this section can be explained with the help of two critical expressions in the model: the free entry condition for posting vacancies (40) and the related definition of the surplus (34). Figure 1 represents the free entry condition as a negative function of vacancies, holding constant the rest of the implied variables. Vacancies enter this expression through the probability of filling a vacancy $\rho_t^f = \vartheta(\frac{u_t}{v_t}, 1)$, whereas changes in other variables shift the curve thus affecting the equilibrium or the impact response and volatility of the vacancy rate. For instance, for a given number of vacancies, an increase in unemployment shifts the curve upwards increasing the number of posted vacancies. Vacancy volatility would in this case depend on the magnitude of the shift, that in turn depends on how much unemployment increased.

Both expressions (40) and (34) contain the main parameters that determine the volatility labor market variables and that have been the subject of much discussion in this literature. The value of non-market activities A and $\widetilde{g}^u$ (inside x_{jt+1}^u) on the one hand, and the bargaining power of workers η, on the other hand, are the key parameters in the calibration discussion for Hagedorn and Manovskii (2005) and Costain and Reiter (2005). Below, we first discuss some calibration issues but mainly focus on a number of economic mechanisms that play an important role in explaining these volatilities for a given set of parameter values. In particular, we study the effects of: (1) endogenous job destruction; (2) intertemporal substitution; (3) price stickiness (i.e. cyclical movements in the mark-up); (4) capital; and (5) taxes.

Endogenous destruction matters through the effect of $\widetilde{a}_{t+1}$. More specifically, the expression (40) can be rewritten in terms of the survival rate $(1\text{-}\rho^x)\,(1\text{-}I\,(\widetilde{a}_{jt}))$ as:

$$\beta E_t\left(\frac{\lambda_{1t+1}}{\lambda_{1t}}\right)\rho_t^f\,(1-\rho^x)\,(1-I\,(\widetilde{a}_{jt}))\int_{\widetilde{a}_{t+1}}^{a_{\max}}(1-\eta)s_{jt+1}^*\frac{\varphi(a)}{(1-I\,(\widetilde{a}_{jt}))}da=\gamma\,(1-\rho^x)$$

A decrease in $\widetilde{a}_{jt}$, as a consequence of a positive shock in productivity, affects the survival rate as well as the average surplus measured by the integral in the above expression. Furthermore, the volatility of vacancies will depend on how much the general equilibrium real interest rate $\frac{\lambda_{1t+1}}{\lambda_{1t}}$ varies after a positive productivity shock. Capital, in turn, enters (34), reducing surplus in levels and therefore making the free entry condition more sensitive to shocks. Taxes affect both the net surplus as well as the dynamics of investment and vacancy posting. Finally, price inertia generates an additional source of surplus dynamics similar to that induced by real wage inertia. Some authors have focused on wage determination as a means of increasing the proportion of the observed volatility of labor market variables that the model is able to explain, while the importance of the price formation mechanism has gone quite unnoticed. Gertler and Trigari, 2005 have looked at the role of wage rigidity, whereas Costain and Reiter (2005) have allowed for countercyclical movements in η. With flexible prices the mark-up $\mu_t=\frac{P_t}{P_t^w}$ is almost constant in presence of technology shocks, with some degree of price stickiness the mark-up increases on impact (due to a fall in P_t^w not compensated by a fall in P_t) and adjusts thereafter thus leading to interesting movements in s_{jt+1}^* that in turn influence current vacancy posting. We investigate these mechanisms in detail below.

4.1 A calibration qualification

The simulation results of the general model described in the previous section appear in the last column of Table 3, as well as the empirical evidence for the United States (first column) and the results for the simplest version of our model, which is comparable to Shimer's (2005). Thus, the model in column (2) is a particular case of the model described in Section 2 that assumes perfect competition in the goods market and price flexibility, with neither capital nor government so that consumption smoothing is not possible and in which job destruction is completely exogenous. In column (2) we present the results of this model using Shimer's calibration for vacancy cost ($\gamma=0.213$), the rate of discount ($1/\beta=1.012$), utility from leisure ($A=0.4$), separation rate ($\rho=0.1$), worker bargaining power ($\eta=0.72$, also equal to the matching elasticity with respect to u) and the scale parameter in the matching function ($\nu_0=1.355$) and choosing the variance and autocorrelation of technology shocks (σ_z and ρ_z) to reproduce these second moments in the case of GDP. As we can see in column (2), we corroborate Shimer's results: the basic search and matching model generates relative volatilities of unemployment and

TABLE 3 — VOLATILITIES
MAIN RESULTS

		US	Basic model Shimer	Basic model	Benchmark
				$A = 0.89$	
		(1)	(2)	(3)	(4)
$\widehat{y}_t$	σ_y	1.58	1.58	1.58	1.58
	ρ_y	0.84	0.84	0.85	0.84
$\ln u_t$	σ_u/σ_y	7.83	0.40	5.74	8.92
	ρ_u	0.87	0.70	0.84	0.83
	$\sigma_{u,y}$	-0.84	-0.83	-0.99	-0.91
$\ln v_t$	σ_v/σ_y	8.85	1.18	4.33	9.72
	ρ_v	0.91	0.70	0.37	0.29
	$\sigma_{v,y}$	0.90	0.97	0.71	0.55
$\ln \frac{v_t}{u_t}$	σ_{vu}/σ_y	16.33	1.46	9.35	14.51
	ρ_{vu}	0.90	0.82	0.67	0.68
	$\sigma_{vu,y}$	0.89	0.99	0.94	0.93
ρ^w	σ_{ρ^w}/σ_y	4.86	0.42	3.23	4.35
	ρ_{ρ^w}	0.91	0.83	0.67	0.69
	$\sigma_{\rho^w,y}$		0.99	0.94	0.94

vacancies which are respectively 20 and 7.5 times smaller than those observed in the data.

In column (3) we present the results for this basic model but with a different calibration. In particular, we choose the same values of parameters as in Table 1, except the utility from leisure ($A = 0.889$, which implies a surplus in steady state equal to 23.2 per cent of output) and the vacancy cost ($\gamma = 0.191$) calibrated to reproduce the same steady state employment and job-finding rates as in Table 2 with the basic search model with exogenous job destruction ($\rho^n = 0$). Both the unemployment rate and vacancies become more volatile than in column (2) with this calibration and the volatility of the v/u ratio also improves accordingly. This result confirms previous findings in the literature (see, for example, Costain and Reiter, 2005, and Hagedorn and Manovskii, 2005) that point out that Shimer's results are somewhat sensitive to calibration. Nevertheless, the volatilities of this basic search and matching model are still far from those observed in the data.

In contrast to the basic model, column (4) offers the main result of the paper: a more realistic model with capital, distortionary taxes and price rigidities can match most dimensions of the cyclical patterns of the US labor market. In particular, the volatility of unemployment, vacancies, and market tightness of the artificial economy are very close to those observed in the data. Notice that in this model the value of A over output is equal to 0.456 in steady state, close to the value used by Shimer (2005).

There are many differences between our general model and Shimer's (2005) making it difficult to gauge the contribution of the different components of the model in explaining the improvement in empirical performance. The rest of this section is devoted to exploring these mechanisms in detail, by taking each of them at a time from the basic to the more general specification. Given the complexity of the model and the absence of an analytical solution this can be only done by relying on numerical simulations and analyzing the sensitivity of the results in each particular case.

4.2 Exogenous versus endogenous job destruction

In Table 4 we present the results of the basic model with (when $\rho^n > 0$) and without endogenous job destruction ($\rho^n = 0$), holding constant all parameters in our model. As we observe, when we introduce endogenous job destruction (that amounts to 1.9 per cent in steady state, representing 21.5 per cent of the total quarterly separation rate) the model predicts higher relative volatility in unemployment but at the cost of a lower volatility in vacancies. This result starkly contrasts with that obtained by Mortensen and Nagypál (2005) who find that endogenous separation increases the volatility of vacancies.

Several effects explain our result. When there is endogenous job destruction and the economy is hit by a positive shock to total factor productivity, the markup does not change (as prices are flexible), whereas the positive innovation in z_t allows firms to operate even under a less favorable firm-specific threshold shock ($\widetilde{a}_{jt}$):

$$\left[\frac{z_t \widetilde{a}_{jt}\left(\widetilde{k}^*_{jt}\right)^{\alpha}}{(1+i_t)\mu_t} - r_t \widetilde{k}^*_{jt}\right] - A + x_{jt} - w^u_{jt} = 0 \tag{62}$$

This reduces the rate of (endogenous) job destruction making the fall in unemployment larger when $\rho^n > 0$. But the larger fall in unemployment (relative to that under exogenous destruction) makes vacancy posting less attractive because the probability of filling a vacancy is lower (i.e. the upwards shift of the left hand side of expression (40) in Figure 1 is lower in this case.

Moreover, the response of the expected surplus is lower under endogenous destruction due to two offsetting mechanisms: first, since the shock in z_t is persistent there is an expected fall in $\widetilde{a}_{jt+1}$ so that matches remain active under worse firm-specific conditions; and second, the probability of a vacancy surviving in the future increases in

TABLE 4 — VOLATILITIES
ENDOGENOUS SEPARATION RATE

		US	Basic model $\rho^n = 0$	Basic model $\rho^n > 0$
		(1)	(2)	(3)
$\widehat{y}_t$	σ_y	1.58	1.58	1.58
	ρ_y	0.84	0.85	0.84
$\ln u_t$	σ_u/σ_y	7.83	5.73	6.56
	ρ_u	0.87	0.84	0.83
	$\sigma_{u,y}$	-0.84	-0.99	-0.99
$\ln v_t$	σ_v/σ_y	8.85	4.33	2.13
	ρ_v	0.91	0.37	0.16
	$\sigma_{v,y}$	0.90	0.71	0.23
$\ln \frac{v_t}{u_t}$	σ_{vu}/σ_y	16.33	9.35	7.42
	ρ_{vu}	0.90	0.67	0.67
	$\sigma_{vu,y}$	0.89	0.94	0.95
ρ^w	σ_{ρ^w}/σ_y	4.86	3.23	2.31
	ρ_{ρ^w}	0.91	0.67	0.67
	$\sigma_{\rho^w,y}$		0.93	0.95

good times. While the first effect diminishes the incentive to post a new vacancy the second does the opposite, but the overall effect is a decrease in $\int_{\widetilde{a}_{t+1}}^{a_{\max}} (1\text{-}\eta) s^*_{jt+1}\varphi(a)da$ thus moderating the increase in current vacancy posting. While Mortensen and Nagypál (2005) only allow in their model for the later effect, thus finding that endogenous destruction increases the volatility of the v/u ratio, in our model endogenous separation creates an additional margin in which the firm can decide without too much of a response from vacancies to technology shocks. Thus the volatility of vacancies falls.

4.3 Intertemporal substitution and price rigidities

We next analyze three key elements in the dynamics of most macroeconomic variables. The matching mechanism is embedded within a more general dynamic model in which agents take their intertemporal decisions by operating through a perfect financial market. We also include habits in consumption as well as price stickiness to enhance the intertemporal nature of decisions taken by households and firms. We first consider Walsh's (2005) model (which is similar to Trigari, 2004), that incorporates intertemporal substitution, price rigidity and habits, but omits capital and distortionary taxes. In column (2) of Table 5 we present the main results in terms of volatilities of this model

TABLE 5 — VOLATILITIES
NOMINAL PRICE RIGIDITIES

		US	Walsh (2005) model			Our model	
			$\omega = 0.7$ $\varsigma = 0.5$	$\omega = 0.05$ $\varsigma = 0$ $\sigma = 1$	$\omega = 0.5$ $\varsigma = 0$	$\omega = 0.5$ $\varsigma = 0$	$\omega = 0.7$ $\varsigma = 0.5$
		(1)	(2)	(3)	(4)	(5)	(6)
$\widehat{y}_t$	σ_y	1.58	1.58	3.27	2.36	2.15	1.58
	ρ_y	0.84	0.94	0.85	0.91	0.77	0.84
$\ln u_t$	σ_u/σ_y	7.83	9.37	10.65	10.04	9.37	8.92
	ρ_u	0.87	0.88	0.80	0.91	0.79	0.83
	$\sigma_{u,y}$	-0.84	-0.94	-0.95	-0.98	-0.97	-0.91
$\ln v_t$	σ_v/σ_y	8.85	6.28	2.93	4.83	6.82	9.72
	ρ_v	0.91	0.46	0.20	0.40	0.12	0.29
	$\sigma_{v,y}$	0.90	0.41	0.23	0.40	0.52	0.55
$\ln \frac{v_t}{u_t}$	σ_{vu}/σ_y	16.33	11.71	11.83	12.56	13.76	14.51
	ρ_{vu}	0.90	0.89	0.69	0.82	0.53	0.68
	$\sigma_{vu,y}$	0.89	0.97	0.91	0.94	0.91	0.93
ρ^w	σ_{ρ^w}/σ_y	4.86	3.61	3.21	3.72	3.96	4.35
	ρ_{ρ^w}	0.91	0.89	0.69	0.81	0.53	0.69
	$\sigma_{\rho^w,y}$		0.97	0.94	0.94	0.92	0.94

using our calibrated parameters, except for σ_z and ρ_z. As we can see, this model does a better job of fitting the relative volatilities of u, v and v/u than the basic model.

Let us first compare models in column (3) in Table 4 and column (3) in Table 5. Both models are similar except for the fact that the latter incorporates the matching mechanism (with endogenous separation) into a standard SDGE framework, as in Walsh (2005) and Trigari (2004) with flexible prices, $\sigma = 1$ and no habits in consumption. The observed difference in volatilities stems from the influence of the intertemporal substitution mechanism. Substituting out the first order conditions of the households into the free entry condition (40) we obtain:

$$E_t \left(\frac{P_{t+1}}{P_t} \frac{1}{1+i_t} \right) \rho_t^f (1-\rho^x) \int_{\widetilde{a}_{t+1}}^{a_{\max}} (1-\eta) s_{jt+1}^* \varphi(a) da = \gamma \tag{63}$$

A positive technology shock induces a fall in the real interest rate that raises the present value of the expected future value of a vacancy, thus inducing a larger upward shift in the left hand side of (63) and more job creation (more vacancies).

Unfortunately intertemporal substitution per se is insufficient for volatilities to approach to those observed and this artificial economy is still far from US data, explaining only $1/3$ of the actual variation in vacancies. This small improvement is not surprising as in this model there is perfect risk sharing among employed and unemployed workers, so the consumption of one particular household is not affected by the employment status. The only difference is at the aggregate level, caused by the response of the real interest rate (i.e. the subjective discount rate) in a model in which households may trade assets in the financial market, an effect that was not present in the models in Table 4.

Two additional mechanisms help to obtain statistics that are closer to those observed in the data. Column (4) of Table 5 depicts the main statistics of a model that includes price rigidity and habits. The presence of habits reinforces the intertemporal substitution mechanism due to the downward reaction of the real interest rate being stronger in the case of habits. When the habit motive is strong ($h = 0.78$), the real interest rate must fall further in order to induce households to advance consumption of the additional output generated by the improvement in technology. Thus strong habits lead to a stronger response of the present value of expected surpluses and, hence, vacancies.

Price rigidity turns out to be the single most important mechanism in explaining the volatility of vacancies. The influence of this channel can be observed by comparing the models in columns (2) and (4) of Table 5 that only differ from each other in the degree of price inertia. In column (4) prices are more flexible and there is less inflation indexation than in column (2) ($\omega = 0.7$, $\varsigma = 0.5$ versus $\omega = 0.5$, $\varsigma = 0$ respectively).

In order to understand this difference we can make use of the entry condition (63). After a positive technology shock the left hand side of (63) shifts upwards, thus increasing the amount of vacancies opened in period t. Apart from the real interest rate, two components of this equation are influenced by the degree of price stickiness in the model. First, the mark-up ($\mu_t = P_t/P_t^w$) increases on impact, due to the downward rigidity of P_t. However μ_{t+1} falls once the progressively downward adjustment of prices is underway. This fall is more intense under sticky prices than under flexible prices and hence the response of s_{t+1}^* is also stronger. Secondly, the technology shock along with the sharp increase in μ_t push $\widetilde{a}_{jt}$ up since the real revenue of intermediate firms is eroded by downward price rigidity in P_t. As a consequence, endogenous destruction rises and unemployment increases. More unemployment reduces labor market tightness, hence increasing the probability of filling a vacancy ρ_t^f when prices are less flexible. These two effects reinforce each other and induce an upward shift on the left hand side of (63) that is larger the higher the degree of price stickiness. Thus the volatility of vacancies and unemployment increase substantially as prices become more rigid.

TABLE 6 — VOLATILITIES
CAPITAL AND DISTORTIONARY TAXES

		US	Benchmark	No capital $\alpha = 0.01$ $\Phi = -10$	No taxes $\tau_t^j = 0\ \forall j$
		(1)	(2)	(3)	(4)
$\widehat{y}_t$	σ_y	1.58	1.58	1.87	1.66
	ρ_y	0.84	0.84	0.92	0.80
$\ln u_t$	σ_u/σ_y	7.83	8.92	10.46	7.69
	ρ_u	0.87	0.83	0.87	0.83
	$\sigma_{u,y}$	-0.84	-0.91	-0.93	-0.92
$\ln v_t$	σ_v/σ_y	8.85	9.72	6.91	9.08
	ρ_v	0.91	0.29	0.51	0.24
	$\sigma_{v,y}$	0.90	0.55	0.44	0.60
$\ln \frac{v_t}{u_t}$	σ_{vu}/σ_y	16.33	14.51	13.17	13.35
	ρ_{vu}	0.90	0.68	0.90	0.62
	$\sigma_{vu,y}$	0.89	0.93	0.96	0.94
ρ^w	σ_{ρ^w}/σ_y	4.86	4.35	3.99	4.10
	ρ_{ρ^w}	0.91	0.69	0.90	0.62
	$\sigma_{\rho^w,y}$		0.94	0.97	0.94

4.4 Capital adjustment costs and distortionary taxes.

A typical assumption in the basic search and matching model is that labor is the only input in the production function. However, as noticed by Mortensen and Nagypál (2005), the elasticity of the v/u ratio with respect to the technology shock increases with the capital share of output. Our definition of the surplus makes it evident why capital affects the net productivity of a job:

$$s_{jt+1}^* = \left[\left(\frac{1}{1+i_{t+1}}\right)\frac{z_{t+1}a_{jt+1}\left(k_{jt+1}^*\right)^\alpha}{\mu_{t+1}} - r_{t+1}k_{jt+1}^*\right] - A + x_{jt+1}^u$$

The response of the surplus is affected by the dynamics of capital and it is therefore worth looking at the adjustment cost of this factor (Φ). We can take Walsh (2005) and Trigari (2004) models as a particular case in which α is small and capital adjustment costs are high, that is, a case in which optimal capital per worker is low and changes very slowly.

In column (3) of Table 6 we depict the effects of a smaller value of α (0.01 versus 0.4) and greater capital adjustment costs (-10 versus -0.25) than those in our benchmark in column (2). First, we observe that the economy with low and persistent capital shows a higher standard deviation of unemployment, since all adjustment after a technology shock relies on labor adjustment. Second, as capital becomes more important in production and adjusts more quickly (column (2)), the response of s^*_{jt+1} to changes in z_t becomes larger making the opening of new jobs more attractive. As a consequence of that, the relative volatility of vacancies increases from 6.91 to 9.72 as we move from one economy in which the capital channel is not relevant (column (3)) to another in which capital is a significant dynamic factor of production (column (2)).[4]

Although of minor importance, the presence of distortionary taxation has also some bearing on the volatility of unemployment and vacancies in our SDGE framework. Distortionary taxes increase the volatility of unemployment since these reduce the after-tax value of the match surplus, thus making firms and workers more selective about the non-production threshold value $\widetilde{a}_{jt}$. The probability of endogenous separation (ρ^n_{jt}) increases more following a positive shock under distortionary taxation and so does unemployment. In addition to this, vacancies are more volatile in this case. First, the increase in unemployment raises the probability of filling a vacancy (ρ^f_t); second, the higher value of $\widetilde{a}_{jt+1}$ increases the expected value of the surplus; and finally, due to the presence of distortionary taxation the steady-state value of capital is lower, so investment reacts more than under lump-sum taxation, and so does s^*_{t+1}.[5] All these effects make posting new vacancies at t more attractive to firms.

5. Concluding Remarks

In the standard search and matching model, the unemployment rate critically hinges upon the number of vacancies posted, which in turn depends on the determinants of the free-entry condition. This condition relates the cost of vacancy posting with the probability of a vacancy being filled as well as with the expected surplus of the vacancy and the discount rate. These three components are model-specific and vary to make vacancy posting more or less responsive to a total factor productivity shock. Shimer (2005) showed that in fact the volatilities of vacancies and unemployment (as well as the

[4] We have also analyzed the sensitivity of our results to changes in Φ holding α constant. As we increase Φ from its benchmark value (-0.25) there is a reduction in the variance of investment, output and vacancies. But this reduction is not uniform, being greater in the case of investment and lower in vacancies than in output. As a result the relative volatility of vacancies to output increases as capital adjustment costs are larger, but at the cost of a smaller relative volatility of investment.

[5] For a thorough discussion on the effects of distortionary taxation (vis-a-vis lump-sum taxes) on the volatility of investment see Andrés and Doménech (2005) and Galí (1994).

vacancy to unemployment ratio) predicted by the basic model are far lower than those observed in US data.

In this paper we hace proposed a more general neo-keynesian dynamic general equilibrium model in which the empirical predictions match the empirical evidence remarkably well. In particular the model predicts a relative (to output) volatility of vacancies, unemployment and the v/u ratio that fit those observed in the data almost perfectly. The model also does well in explaining autocorrelations and cross correlations among variables, although the implied persistence of vacancies is somewhat low.

Since the model has a complex structure, we carry out a detailed analysis of the contribution of each of the main features of the model to this enhanced empirical performance. Endogenous separation actually takes model predictions further away from those observed since it opens up a channel through which firms can absorb the effects of a positive technology shock without relying too much on new vacancy posting. All determinants of the free entry condition and, therefore, of vacancy posting, are crucially affected by the introduction of consumption smoothing, habits, price inertia, capital and distortionary taxes. As the model allows for consumption smoothing and higher habits, the discount rate falls more (than would have been the case in an economy without these features) following a positive technology shock. Furthermore, price inertia and distortionary taxes affect the matching specific threshold productivity level and the value of future surpluses resulting in a increase in the expected value of vacancies. Finally the presence of capital also strengthens the response of expected surpluses to technology shocks. All these model features contribute to increase the attractiveness of new vacancy posting following a positive innovation to total factor productivity, thus taking the model predictions about the volatility of vacancies, unemployment and the vacancy to unemployment ratio to levels comparable with the empirical evidence.

One of these mechanisms turns out to be of paramount importance: price stickiness. It has a direct effect on all components of the free entry condition and has proved to be the most significant in quantitative terms. In this sense, we see our results to be akin to those emphasizing the importance of nominal stickiness, mostly in nominal wages, as a way of improving the empirical performance of matching models. The combination of wage and price stickiness seems a natural extension aimed at both to further improving the model and also assessing the relative importance of different sources of nominal inertia for the purpose at hand.

A final comment on calibration is pertinent here. Our empirical analysis has been ushered in by a thorough calibration exercise based on a careful analysis of the existing literature on the issue, as well as of the basic steady-state variables for the US economy. The empirical predictions of our general model are fairly robust to reasonable changes in calibration values. However, we have also corroborated that the predictions of the basic Mortensen and Pissarides model were somewhat sensitive to the choice of parameter values. In fact the simulated moments for some parameterization, although clearly below target, were not as far away from the empirical observation as the previous literature suggested. This leads us to believe that more research is needed on this matter and in particular a deep econometric analysis is called for to obtain a better empirical counterpart of some of the parameters used in this literature. This is next on the research agenda.

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Appendix 1: Equilibrium

The dynamic equilibrium is defined by the following equations:

$$y_t = \frac{(1-\rho_t) n_t (1+i_t) \mu_t r_t}{\alpha} k_t^* - \gamma v_t \tag{1.1}$$

$$c_t + e_t + g_t^c = y_t \tag{1.2}$$

$$\lambda_{1t} = \frac{\frac{c_t^{-\sigma}}{c_{t-1}^{h(1-\sigma)}} - \beta E_t h \frac{c_{t+1}^{1-\sigma}}{c_t^{h(1-\sigma)+1}}}{(1+\tau_t^c)(1+i_{t-1})} \tag{1.3}$$

$$\lambda_{1t}\beta^{-1} = (1+i_t) E_t \left(\lambda_{1t+1} \frac{P_t}{P_{t+1}} \right) \tag{1.4}$$

$$P_t (1+\tau_t^c) c_t = M_t \tag{1.5}$$

$$k_t = (1-\delta) k_{t-1} + \phi \left(\frac{e_t}{k_{t-1}} \right) k_{t-1} \tag{1.6}$$

$$\left[\phi' \left\{ \frac{e_t}{k_{t-1}} \right\} \right]^{-1} = q_t \tag{1.7}$$

$$q_t \beta^{-1} = E_t \left[\frac{\lambda_{1t+1}}{\lambda_{1t}} \left(\begin{array}{c} (1-\tau_t^k) r_{t+1} + \\ q_{t+1} \left[(1-\delta) + \phi \left\{ \frac{e_{t+1}}{k_t} \right\} - \phi' \left\{ \frac{e_{t+1}}{k_t} \right\} \frac{e_{t+1}}{k_t} \right] \end{array} \right) \right] \tag{1.8}$$

$$P_t^* = \left(\frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t,t+s} \left[\mu_{t+s}^{-1} (P_{t+s})^{\theta+1} c_{t+s} \right]}{E_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t,t+s} \left[(P_{t+s})^{\theta} c_{t+s} \right]} \tag{1.9}$$

$$P_t^{1-\theta} = (1-\omega) P_t^{*1-\theta} + \omega P_{t-1}^{1-\theta} \tag{1.10}$$

$$k_{jt}^{*}=\left(\frac{\alpha z_{t}a_{jt}}{\left(1+i_{t}\right)\mu_{t}r_{t}}\right)^{\frac{1}{1-\alpha}} \tag{1.11}$$

$$\left[\frac{z_{t}\widetilde{a}_{t}\left(\widetilde{k}_{t}^{*}\right)^{\alpha}}{\left(1+i_{t}\right)\mu_{t}}-r_{t}\widetilde{k}_{t}^{*}\right]-A+x_{t}^{u}=0 \tag{1.12}$$

$$\rho_{t}^{n}=\int_{-\infty}^{\widetilde{a}_{t}}\varphi(a_{t})da \tag{1.13}$$

$$\rho_{t}=\rho^{x}+\left(1-\rho^{x}\right)\rho_{t}^{n} \tag{1.14}$$

$$\rho_{t}^{s}=1-\rho_{t} \tag{1.15}$$

$$s_{jt+1}^{*}=\left[\left(\frac{1}{1+i_{t+1}}\right)\frac{z_{t+1}a_{jt+1}\left(k_{jt+1}^{*}\right)^{\alpha}}{\mu_{t+1}}-r_{t+1}k_{jt+1}^{*}\right]-A+x_{t+1}^{u} \tag{1.16}$$

$$s_{t+1}^{*}=\frac{1-\alpha}{\alpha}r_{t+1}k_{t+1}^{*}-A+x_{t+1}^{u} \tag{1.17}$$

$$x_{t}^{u}=\beta E_{t}\left(\frac{\lambda_{1t+1}}{\lambda_{1t}}\right)\left(1-\rho_{t+1}\right)\left[1-\eta\rho_{t}^{w}-\eta\tau_{t+1}^{w}\left(1-\rho_{t}^{w}\right)\right]s_{t+1}^{*}-\widetilde{g}^{u} \tag{1.18}$$

$$x_{t}^{u}=\frac{\gamma\left[1-\eta\rho_{t}^{w}-\eta\tau_{t+1}^{w}\left(1-\rho_{t}^{w}\right)\right]}{\rho_{t}^{f}\left(1-\eta\right)}-\widetilde{g}^{u} \tag{1.19}$$

$$y_{t}^{l}=(1-\rho_{t})n_{t}\left(1-\tau_{t}^{w}\right)\left[\eta s_{t}^{*}+A-x_{t}^{u}\right] \tag{1.20}$$

$$u_{t}=1-\left(1-\rho_{t}\right)n_{t} \tag{1.21}$$

$$\rho_t^w = \frac{\vartheta(u_t, v_t)}{u_t} \tag{1.22}$$

$$\rho_t^f = \frac{\vartheta(u_t, v_t)}{v_t} \tag{1.23}$$

$$n_{t+1} = (1 - \rho_t) n_t + \vartheta(u_t, v_t) \tag{1.24}$$

$$(1 - \rho_t)\, n_t k_t^* = k_{t-1} \tag{1.25}$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[\rho_\pi(\pi_t - \overline{\pi_t}) + \rho_y \left(\widehat{y}_t\right) + \overline{i}\right] \tag{1.26}$$

$$t_t = \tau_t^c c_t + \tau_t^k r_t k_{t-1} + \tau_t^w (1 - \rho_t) n_t \left(\eta s_t^* + A - x_t^u\right) \tag{1.27}$$

$$b_t - (1 + i_{t-1}) \frac{b_{t-1}}{\pi_t} = g_t^c + g_t^s + g^u u_t - t_t \tag{1.28}$$

$$g_t^\varphi = g_{t-1}^\varphi + \psi_1^\varphi \left[\overline{\left(\frac{b}{y}\right)} - \left(\frac{b_t}{y_t}\right)\right] + \psi_2^\varphi \left[\left(\frac{b_{t-1}}{y_{t-1}}\right) - \left(\frac{b_t}{y_t}\right)\right] \tag{1.29}$$

$$\frac{E_t \Lambda_{t,t+s}}{E_t \Lambda_{t,t+s-1}} = \frac{E_t(\lambda_{1t+s}/P_{t+s})}{E_t(\lambda_{1t+s-1}/P_{t+s-1})} \tag{1.30}$$

$$k_t^* = \int_{\widetilde{a}_t}^{a_{\max}} k_{jt}^* \frac{\varphi(a)}{1 - \Phi(\widetilde{a})} da = \tag{31}$$
$$\left(\frac{\alpha z_t}{(1 + i_t)\, \mu_t r_t}\right)^{\frac{1}{1-\alpha}} \int_{\widetilde{a}_t}^{a_{\max}} \frac{a_t^{\frac{1}{1-\alpha}} \varphi(a)}{1 - I(\widetilde{a}_t)} da$$

$$\widetilde{k}_t^* = \left(\frac{\alpha z_t \widetilde{a}_t}{(1+i_t)\,\mu_t r_t} \right)^{\frac{1}{1-\alpha}} \tag{1.32}$$

$$\pi_t = \frac{P_{t+1}}{P_t} \tag{1.33}$$

Endogenous variables:c_t, e_t, y_t, λ_{1t}, i_t, r_t, v_t, u_t, $\widetilde{a}_t$, n_t, k_{jt}^*, π_t, M_t, P_t, q_t, P_t^*, Λ_t, μ_t, x_t^u, ρ_t^n, ρ_t, ρ_t^w, ρ_t^f, ρ_t^s, t_t, b_t, g_t^φ, k_t, y_t^l, k_t^*, $\widetilde{k}_t^*$,.s_{jt+1}^*, s_{t+1}^*
(33 equations=33 variables)

Appendix 2: The steady-state model

From (1.21):

$$\overline{u} = 1 - (1-\overline{\rho})\,\overline{n} \tag{2.1}$$

From (1.24):

$$\overline{\rho n} = \vartheta(\overline{u}, \overline{v}) \tag{2.2}$$

From (1.22):

$$\overline{\rho}^w = \frac{\vartheta(\overline{u}, \overline{v})}{\overline{u}} \tag{2.3}$$

From (1.23):

$$\overline{\rho}^f = \frac{\vartheta(\overline{u}, \overline{v})}{\overline{v}} \tag{2.4}$$

From (1.13) and (1.14):

$$\overline{\rho} = \rho^x + (1-\rho^x)\, I\left(\widetilde{a}\right) \tag{2.5}$$

From (1.15):

$$\overline{\rho}^s = 1 - \overline{\rho} \tag{2.6}$$

From (1.4):

$$\beta = \frac{\overline{\pi}}{1+\overline{i}} \tag{2.7}$$

From (1.8):

$$\overline{q}\beta^{-1} = \left(\left(1 - \overline{\tau}^{k}\right)\overline{r} + \overline{q}\left[(1-\delta) + \phi\left\{\frac{\overline{e}}{\overline{\overline{k}}}\right\} - \phi'\left\{\frac{\overline{e}}{\overline{k}}\right\}\frac{\overline{e}}{\overline{\overline{k}}}\right]\right) \tag{2.8}$$

From (1.7):

$$\left[\phi'\left\{\frac{\overline{e}}{\overline{\overline{k}}}\right\}\right]^{-1} = \overline{q} \tag{2.9}$$

From (1.9):

$$\left(\frac{\theta}{\theta - 1}\right) = \overline{\mu} \tag{2.10}$$

From (1.31):

$$\overline{k}^{*} = \frac{1}{\left(1 - I\left(\overline{\overline{a}}\right)\right)}\left(\frac{\alpha}{\left(1+\overline{i}\right)\overline{\mu r}}\right)^{\frac{1}{1-\alpha}}\int_{\overline{\overline{a}}}^{a_{\max}} a^{\frac{1}{1-\alpha}}\varphi(a)da \tag{2.11}$$

From (1.25):

$$(1-\rho)\,\overline{n}\overline{k}^{*} = \overline{k} \tag{2.12}$$

From (1.32):

$$\overline{\overline{k}}^{*} = \left(\frac{\alpha\overline{\overline{a}}}{\left(1+\overline{i}\right)\overline{\mu r}}\right)^{\frac{1}{1-\alpha}} \tag{2.13}$$

From (1.18)[6]:

$$\overline{x}^u = \beta\left(1-\overline{\rho}\right)\left[1-\eta\overline{\rho}^w-\eta\overline{\tau}^w\left(1-\overline{\rho}^w\right)\right]\overline{s}^* - \left(1-\beta^2\left[1-\rho^w\left(1-\rho^x\right)\right]^2\right)g^u \quad (2.14)$$

From (1.12):

$$\overline{x}^u = A - \left[\frac{\overline{\widetilde{a}}\left(\overline{\widetilde{k}}^*\right)^\alpha}{\left(1+\overline{i}\right)\overline{\mu}} - \overline{r}\overline{\widetilde{k}}^*\right] \quad (2.15)$$

From (1.17):

$$\overline{s}^* = \frac{1-\alpha}{\alpha}\overline{r}\overline{k}^* - A + \overline{x}^u \quad (2.16)$$

From (1.19):

$$A - \left(\frac{\widetilde{a}\overline{\widetilde{k}}^{*\alpha}}{\overline{\mu}\left(1+\overline{i}\right)} - \overline{r}\overline{\widetilde{k}}^*\right) = \frac{\gamma\left[1-\overline{\rho}^w\eta-\overline{\overline{\tau}}^w\eta\left(1-\overline{\rho}^w\right)\right]}{\left(1-\eta\right)\overline{\rho}^f} - \left(1-\beta^2\left[1-\rho^w\left(1-\rho^x\right)\right]^2\right)g^u \quad (17)$$

From (1.1):

$$\overline{y} = \frac{(1-\overline{\rho})\overline{n}\left(1+\overline{i}\right)\overline{\mu r}}{\alpha}\overline{k}^* - \gamma\overline{v} \quad (2.18)$$

From (1.6):

$$\frac{\overline{e}}{\overline{\overline{k}}} = \phi^{-1}\left(\delta\right) \quad (2.19)$$

[6] From 37 can be obtained the steady-state expected present value of income coming from $\widetilde{g}^u$ as:

$$\left[1+\beta\left(1-\rho^w\left(1-\rho^x\right)\right)+\beta^2\left(1-\rho^w\left(1-\rho^x\right)\right)^2+\beta^3\left(1-\rho^w\left(1-\rho^x\right)\right)^3....\right]\widetilde{g}^u$$

We wish to calibrate $\widetilde{g}^u$ so that the observed unemployment benefits (g^u) is received only during two consecutive periods:

$$\left[1+\beta\left(1-\rho^w\left(1-\rho^x\right)\right)\right]g^u = \left[\frac{1}{1-\beta\left(1-\rho^w\left(1-\rho^x\right)\right)}\right]\widetilde{g}^u$$

Therfore

$$\widetilde{g}^u = \left(1-\left[\beta\left(1-\rho^w\left(1-\rho^x\right)\right)\right]^2\right)g^u$$

From (1.2):

$$\overline{c} + \overline{e} + \overline{g}^c = \overline{y} \tag{2.20}$$

From (1.3):

$$\left(1 + \overline{\tau}^c\right)\left(1 + \overline{i}\right)\overline{\lambda}_1 = (1 - \beta h)\,\frac{\overline{c}^{\sigma(h-1)}}{\overline{c}^h} \tag{2.21}$$

From (1.5):

$$\left(1 + \overline{\tau}^c\right)\overline{c} = \frac{\overline{M}}{\overline{P}} \tag{2.22}$$

From (1.20):

$$\overline{y}^l = (1 - \overline{\rho})\,\overline{n}\,(1 - \overline{\tau}^w)\left[\eta\overline{s}^* + A - \overline{x}^u\right] \tag{2.23}$$

From (1.27):

$$\overline{t} = \overline{\tau}^c\overline{c} + \overline{\tau}^k r_t \overline{k} + \overline{\tau}^w (1 - \overline{\rho})\,\overline{n}\left(\eta\overline{s}^* + A - \overline{x}^u\right) \tag{2.24}$$

From (1.28):

$$\overline{g}^c + \overline{g}^s + g^u\overline{u} + \overline{i}\overline{b} = \overline{t} \tag{2.25}$$

Exogenous variables: $\overline{\pi}$ and $\overline{\tau}^c$, $\overline{\tau}^k$, $\overline{\tau}^w$, $\overline{g}^c$, $\overline{g}^s$, $\overline{g}^u$. Endogenous:$\overline{c}$, $\overline{e}$, $\overline{y}$, $\overline{\lambda}_1$, $\overline{i}$, $\overline{r}$, $\overline{v}$, $\overline{u}$, $\widetilde{a}$, $\overline{n}$, $\overline{m}$, $\overline{q}$, $\overline{\mu}$, $\overline{x}^u$, $\overline{\rho}$, $\overline{s}^*$, $\overline{\rho}^w$, $\overline{\rho}^f$, $\overline{y}^l$, $\overline{t}$, $\overline{b}$, $\overline{k}$, $\overline{\widetilde{k}}^*$, $\overline{k}^*$, $\overline{\rho}^s$ (25 endogenous=25 equations)

Appendix 3: Log-linearized model

Let $\widehat{x}$ be the variable that tell us how much x differs from its steady-state value.

From (1.12):

$$\begin{aligned}
\widehat{\widetilde{a}}_t \quad = \quad & \left(\frac{\overline{i}}{1 + \overline{i}}\right)\widehat{i}_t - \widehat{z}_t + \widehat{\mu}_t - \left(\alpha - \frac{\overline{r}\overline{\widetilde{k}}^*}{\overline{r}\overline{\widetilde{k}}^* + A - \overline{x}^u}\right)\widehat{\widetilde{k}}_t^* \\
& + \left(\frac{\overline{r}\overline{\widetilde{k}}^*}{\overline{r}\overline{\widetilde{k}}^* + A - \overline{x}^u}\right)\widehat{r}_t - \left(\frac{\overline{x}^u}{\overline{r}\overline{\widetilde{k}}^* + A - \overline{x}^u}\right)\widehat{x}_t^u
\end{aligned} \tag{3.1}$$

From (1.13):

$$\widehat{\rho}_t^n = \frac{\varphi(\overline{\widetilde{a}})\overline{\widetilde{a}}}{I\left(\overline{\widetilde{a}}\right)}\widehat{\widetilde{a}}_t \tag{3.2}$$

From (1.14):

$$\widehat{\rho}_t = \left[\frac{(1-\rho^x)\,\overline{\rho}^n}{\overline{\rho}}\right]\widehat{\rho}_t^n \tag{3.3}$$

From (1.15):

$$\widehat{\rho}_t^s = \frac{-\overline{\rho}}{1-\overline{\rho}}\widehat{\rho}_t \tag{3.4}$$

From (1.24):

$$\widehat{n}_{t+1} = (1-\overline{\rho})\widehat{n}_t - \overline{\rho}\widehat{\rho}_t + \overline{\rho}^w \frac{\overline{v}^\nu}{\overline{u}^\nu + \overline{v}^\nu}\frac{\overline{u}}{\overline{n}}\widehat{u}_t + \overline{\rho}^f \frac{\overline{u}^\nu}{\overline{u}^\nu + \overline{v}^\nu}\frac{\overline{v}}{\overline{n}}\widehat{v}_t \tag{3.5}$$

From (1.21):

$$\widehat{u}_t = -\left(1-\overline{\rho}\right)\frac{\overline{n}}{\overline{u}}\widehat{n}_t + \overline{\rho}\frac{\overline{n}}{\overline{u}}\widehat{\rho}_t \tag{3.6}$$

From (1.23):

$$\widehat{\rho}_t^f = \frac{\overline{v}^\nu}{\overline{u}^\nu + \overline{v}^\nu}\left(\widehat{u}_t - \widehat{v}_t\right) \tag{3.7}$$

From (1.22):

$$\widehat{\rho}_t^w = \frac{\overline{u}^\nu}{\overline{u}^\nu + \overline{v}^\nu}\left(\widehat{v}_t - \widehat{u}_t\right) \tag{3.8}$$

From (1.19):

$$\widehat{\rho}_t^f = -\frac{\eta\overline{\rho}^w\left(1-\overline{\tau}^w\right)}{1-\eta\overline{\rho}^w\left(1-\overline{\tau}^w\right)-\eta\overline{\tau}^w}\widehat{\rho}_t^w - \frac{\overline{x}_t^u}{\overline{x}_t^u + \widetilde{g}^u}\widehat{x}_t^u \tag{3.9}$$

From (1.1):

$$\widehat{y}_t = \left(\frac{\overline{y}+\gamma\overline{v}}{\overline{y}}\right)\left[\widehat{n}_t - \left(\frac{\overline{\rho}}{1-\overline{\rho}}\right)\widehat{\rho}_t + \left(\frac{\overline{i}}{1+\overline{i}}\right)\widehat{i}_t + \widehat{\mu}_t + \widehat{r}_t + \widehat{k}_t^*\right] - \frac{\gamma\overline{v}}{\overline{y}}\widehat{v}_t \tag{3.10}$$

From (1.18):

$$\begin{aligned}\widehat{x}_t^u \quad = \quad & \left(\frac{\overline{x}^u+\widetilde{g}^u}{\overline{x}^u}\right)\left(E_t\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + E_t\widehat{s}_{t+1}^*\right) - \\ & \left(\frac{\beta\overline{s}\eta\overline{\rho}^w}{\overline{x}^u}\right)(1-\overline{\tau}^w)(1-\overline{\rho})\widehat{\rho}_t^w - \left(\frac{\beta\overline{s}\overline{\rho}}{\overline{x}^u}\right)\left(1-\eta\overline{\rho}^w-\eta\overline{\tau}^w(1-\overline{\rho}^w)\right)E_t\widehat{\rho}_{t+1}\end{aligned} \tag{3.11}$$

From (1.17):

$$\widehat{s}_t^* = \left(\frac{1-\alpha}{\alpha}\right)\frac{\overline{r}\overline{k}^*}{\overline{s}^*}\left(\widehat{k}_t^* + \widehat{r}_t\right) + \frac{\overline{x}^u}{\overline{s}^*}\widehat{x}_t^u \tag{3.12}$$

From (1.2):

$$\widehat{y}_t = \frac{\overline{c}}{\overline{y}}\widehat{c}_t + \frac{\overline{e}}{\overline{y}}\widehat{e}_t + \frac{\overline{g}}{\overline{y}}\widehat{g}_t^c \tag{3.13}$$

From (1.4):

$$\widehat{\lambda}_{1t} = \frac{\overline{i}}{\left(1+\overline{i}\right)}\widehat{i}_t + E_t\left(\widehat{\lambda}_{1t+1} - \widehat{\pi}_{t+1}\right) \tag{3.14}$$

From (1.5):

$$\widehat{M}_t = \widehat{P}_t + \widehat{c}_t \tag{3.15}$$

From (1.6):

$$\widehat{k}_t = \left(1 - \frac{\overline{e}}{\overline{k}}\right)\widehat{k}_{t-1} + \frac{\overline{e}}{\overline{k}}\widehat{e}_t \tag{3.16}$$

From (1.7):

$$\widehat{q}_t = \phi''\frac{\overline{e}}{\overline{k}}\left(\widehat{k}_{t-1} - \widehat{e}_t\right) \tag{3.17}$$

From (1.8):

$$\begin{aligned}\widehat{q}_t \quad = \quad & E_t\left(\widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t}\right) + \beta\overline{r}\left(1-\overline{\tau}^k\right)E_t\widehat{r}_{t+1} + \\ & \beta\left(1-\frac{\overline{e}}{\overline{k}}\right)E_t\widehat{q}_{t+1} - \beta\left(\frac{\overline{e}}{\overline{k}}\right)^2\phi''E_t\left(\widehat{e}_{t+1} - \widehat{k}_t\right)\end{aligned} \tag{3.18}$$

From (1.10):

$$E_t\widehat{P}_{t+1}^* = \frac{1}{(1-\omega)}E_t\left(\widehat{P}_{t+1} - \widehat{P}_t\right) + \widehat{P}_t \tag{3.19}$$

From (1.26):

$$\widehat{\overline{i}i}_t = \rho_i\widehat{\overline{i}i}_{t-1} + (1-\rho_i)\,\rho_\pi\overline{\pi}\widehat{\pi}_t + (1-\rho_i)\,\rho_y\overline{y}\widehat{y}_t \tag{3.20}$$

Fom (1.9):

$$\widehat{P}_t^* = \beta\omega E_t\widehat{P}_{t+1}^* + (1-\beta\omega)\left(\widehat{P}_t - \widehat{\mu}_t\right) \tag{3.21}$$

From (1.30):

$$E_t\widehat{\Lambda}_{t+1} = \widehat{\Lambda}_t + E_t\left(\widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t}\right) - E_t\left(\widehat{P}_{t+1} - \widehat{P}_t\right) \tag{3.22}$$

From (1.3):

$$\begin{aligned}\widehat{\lambda}_{1t} &= \frac{\beta h(1+h(1-\sigma))-\sigma}{1-\beta h}\widehat{c}_t - \frac{h(1-\sigma)}{1-\beta h}\widehat{c}_{t-1} \\ &\quad -\frac{\beta h(1-\sigma)}{1-\beta h}E_t\widehat{c}_{t+1} - \frac{\bar{\bar{i}}}{1+\bar{\bar{i}}}\widehat{i}_{t-1}\end{aligned} \tag{3.23}$$

From (1.20):

$$\begin{aligned}\widehat{y}_t^l &= \widehat{n}_t - \left(\frac{\overline{\rho}}{1-\overline{\rho}}\right)\widehat{\rho}_t \\ &\quad + \frac{\eta(1-\rho)\overline{n}\left(1-\overline{\tau}^w\right)s}{\overline{y}^l}\widehat{s}_t^* - \frac{(1-\rho)\overline{n}\left(1-\overline{\tau}^w\right)\overline{x}^u}{\overline{y}^l}\widehat{x}_t^u\end{aligned} \tag{3.24}$$

From (1.25):

$$\widehat{k}_{t-1} = \widehat{n}_t - \frac{\overline{\rho}}{1-\overline{\rho}}\widehat{\rho}_t + \widehat{k}_t^* \tag{3.25}$$

New Phillips curve:

$$\widehat{\pi}_t = \frac{\beta}{1+\varsigma\beta}E_t\widehat{\pi}_{t+1} - \frac{(1-\beta\omega)(1-\omega)}{\omega(1+\varsigma\beta)}\widehat{\mu}_t + \frac{\varsigma}{1+\varsigma\beta}\widehat{\pi}_{t-1} \tag{3.26}$$

From (1.27):

$$\begin{aligned}\widehat{t}_t &= \frac{\overline{\tau}^c\overline{c}}{\overline{t}}\widehat{c}_t + \frac{\overline{\tau}^k\overline{r}\overline{k}}{\overline{t}}\left(\widehat{k}_{t-1} + \widehat{r}_r\right) + \frac{\overline{\tau}^w\overline{n}(1-\overline{\rho})}{\overline{t}}\left(\eta\overline{s}^* + A - \overline{x}^u\right)\left(\widehat{n}_t - \frac{\overline{\rho}}{(1-\overline{\rho})}\widehat{\rho}_t\right) \\ &\quad + \frac{\overline{\tau}^w\overline{n}(1-\overline{\rho})\eta\overline{s}^*}{\overline{t}}\widehat{s}_t^* - \frac{\overline{\tau}^w\overline{n}(1-\overline{\rho})\overline{x}^u}{\overline{t}}\widehat{x}_t^u\end{aligned} \tag{3.27}$$

From (1.29):

$$\overline{g}^\varphi\widehat{g}_t^\varphi = \overline{g}^\varphi\widehat{g}_{t-1}^\varphi + \left(\frac{\overline{b}}{y}\right)\left(\psi_1^\varphi + \psi_2^\varphi\right)\left(\widehat{y}_t - \widehat{b}_t\right) + \psi_2^\varphi\left(\frac{\overline{b}}{y}\right)\left(\widehat{b}_{t-1} - \widehat{y}_{t-1}\right) \tag{3.28}$$

From (1.28):

$$\overline{t}\widehat{t}_t = \overline{g}^c\widehat{g}_t^c + \overline{g}^s\widehat{g}_t^s + g^u\overline{u}\widehat{u}_t + \frac{\overline{b}}{\overline{\pi}}\overline{i}\widehat{i}_{t-1} - \frac{\overline{b}}{\overline{\pi}}\left(1+\overline{i}\right)\widehat{\pi}_t + \frac{\overline{b}}{\overline{\pi}}\left(1+\overline{i}\right)\widehat{b}_{t-1} - \overline{b}\widehat{b}_t \tag{3.29}$$

From (1.31):

$$\widehat{k}_t^* = \left(\frac{1}{1-\alpha}\right)\left(\widehat{z}_t - \frac{\bar{i}}{1+\bar{i}}\widehat{i}_t - \widehat{\mu}_t - \widehat{r}_t\right) - \Psi(\bar{\widetilde{a}})\widehat{\widetilde{a}}_t \tag{3.30}$$

where:

$$\Psi(\bar{\widetilde{a}}) = \bar{\widetilde{a}}\varphi(\bar{\widetilde{a}})\left[\frac{1}{1-I\left(\bar{\widetilde{a}}\right)} - \frac{\left(\bar{\widetilde{a}}\right)^{\left(\frac{1}{1-\alpha}\right)}}{\int_{\bar{\widetilde{a}}}^{a_{\max}}(a)^{\left(\frac{1}{1-\alpha}\right)}\varphi(a)da}\right] \tag{3.31}$$

From (1.32):

$$\widehat{\widetilde{k}}_t^* = \left(\frac{1}{1-\alpha}\right)\left(\widehat{z}_t + \widehat{\widetilde{a}}_t - \frac{\bar{i}}{1+\bar{i}}\widehat{i}_t - \widehat{\mu}_t - \widehat{r}_t\right) \tag{3.32}$$